

Original Research

Graph-Based Latent Dynamical Learning for Distributed Sensor Networks under Partial Observability and Communication Constraints

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Abstract

Distributed sensor networks are increasingly deployed in environments where the underlying physical process is high-dimensional, nonlinear, spatially coupled, and only partially observable through noisy local measurements. In such settings, useful inference depends not only on local prediction accuracy but also on the ability to recover coherent global structure from sparse asynchronous observations, preserve stability over long temporal horizons, and operate within communication and computation limits imposed by decentralized hardware. These requirements are difficult to satisfy jointly because spatial dependence, temporal nonstationarity, sensor heterogeneity, and network failures interact in ways that degrade both estimation quality and operational reliability. This paper develops a graph-based latent dynamical framework for distributed sensor networks that integrates spatial topology, latent-state evolution, uncertainty-aware inference, and communication-efficient learning within a common mathematical formulation. The model represents local measurements through graph-coupled latent variables whose dynamics evolve under a structured operator combining diffusion, low-rank global transport, and temporally adaptive residual terms. The resulting formulation supports filtering, smoothing, forecasting, anomaly sensitivity, and network-level state reconstruction while preserving scalability through decentralized optimization and low-dimensional message exchange. The paper analyzes the geometry of the latent representation, the spectral conditioning of the learned transition operators, and the effect of graph regularization on stability and identifiability under missing data and topology perturbations. It also develops a rigorous evaluation protocol for studying adaptation under drift, communication bottlenecks, and sensor faults. The overall perspective emphasizes that reliable learning in distributed sensing systems requires coordinated treatment of latent representation, dynamical structure, and network-aware computation.

1. Introduction

Distributed sensor networks have become a central substrate for modern environmental monitoring, industrial inspection, intelligent transportation, structural health analysis, energy systems, and autonomous infrastructure management [1]. Their appeal lies in the ability to obtain broad spatial coverage through many modest sensing devices rather than relying on a small number of high-capability stations. Yet the same feature that makes these systems attractive also makes them difficult to model. Measurements arrive from many locations with different sampling rates, unequal reliability, varying calibration histories [2], and partial communication connectivity. The underlying process generating the measurements may exhibit diffusion, transport, local reaction, exogenous forcing, abrupt shocks, or slow regime drift. No single sensor sees the entire state, and the network as a whole often observes the physical field only through a sparse and noisy projection. As a result, learning in distributed sensor networks is not simply a matter of fitting independent time series or applying a static graph predictor. It

requires a representation that is at once local and global, expressive and stable, adaptive and computationally efficient. A recurring difficulty in this domain is the mismatch between the structure of the physical process and the structure of the learned model [3]. If one treats each node independently, the model ignores the fact that neighboring sensors often reflect the same latent transport or diffusion mechanism. If one imposes a rigid graph smoothness prior, the model may oversuppress localized events, faults, or boundaries that carry operational significance. If one trains a deep recurrent architecture without explicit structural constraints, the model can fit short horizons but remain fragile under recursive rollout, especially when communication delays or sensor dropout alter the effective data distribution during deployment. These issues become more pronounced when the graph itself is imperfect, time-varying, or only partially aligned with the latent field dynamics. In practice, the challenge is not only to predict the next observation. The challenge is to recover a coherent low-dimensional state that can summarize the evolving field, propagate uncertainty, support anomaly detection, and remain numerically stable as new data arrive and local conditions change [4].

The graph-based latent dynamical viewpoint is attractive precisely because it provides a language for linking spatial organization with temporal inference. A graph encodes which sensors are likely to exchange information, share common forcing, or participate in related physical interactions. A latent representation compresses the high-dimensional and noisy observation field into a structured state whose temporal evolution can be analyzed, regularized, and estimated. The combination is especially useful when the true system admits multiple scales of variation. Slow graph-smooth modes may describe broad environmental trends, while sharper localized modes represent shocks, obstructions, failures, or transient disturbances. A useful model should preserve both, but not indiscriminately [5]. It should retain dynamically meaningful modes while damping unsupported fluctuations that would otherwise amplify under recursive forecasting or decentralized optimization.

This perspective aligns with broader concerns in structured learning for dynamical systems, including the need to regularize latent evolution in ways that preserve stability, adaptability, and operational interpretability, as discussed by Hasan et al. (2024) [6]. The present paper develops that general philosophy for the specific setting of distributed sensor networks, where graph structure, communication limits, and partial observability jointly shape what can be inferred and how learning must be organized across the network.

The core idea of the paper is to construct a latent state sequence whose evolution is governed by a graph-aware operator family rather than by an unconstrained black-box recurrence. Each time step is associated with a latent field over the network, and that field evolves through a transition operator composed of graph diffusion, low-rank global transport, and an adaptive correction term that can absorb regime shifts or local structural changes. The encoder maps raw local observations into latent coordinates, the dynamic operator propagates those coordinates across time, and the decoder reconstructs network-level state estimates or predicted measurements. The graph is not used merely as an input adjacency matrix for message passing; it is integrated into the latent geometry itself through spectral regularization and consensus-aware estimation. This distinction matters because the geometry of the latent space determines how uncertainty, communication noise, and model mismatch accumulate over time.

Another important aspect of the framework is that it treats decentralized computation as part of the modeling problem rather than as an implementation detail. In realistic deployments, sensors or edge processors cannot continually transmit full raw histories to a central server. Communication may be energy-limited, delayed, intermittent, or expensive relative to local computation. Consequently, a viable inference framework must support local updates, compressed message exchange, and consensus formation over a sparse connectivity graph. This requirement changes the character of the learning problem. It is not enough for a model to be statistically efficient in centralized batch form. It must also admit local sufficient statistics, stable distributed optimization, and graceful degradation under reduced bandwidth or temporary disconnection.

The paper therefore pursues several linked objectives. The first is to define a graph-based latent state model that preserves both spatial and temporal structure without forcing a brittle global stationarity assumption. The second is to derive inference and learning procedures that can operate sequentially

under partial observation and limited communication. The third is to analyze the stability, identifiability, and spectral behavior of the resulting operator family, since these properties determine whether the model can support long-horizon forecasting and reliable reconstruction [7]. The fourth is to characterize robustness under missing data, faulty sensors, and topology perturbations. Rather than treating these as separate application-specific add-ons, the paper incorporates them into the formal problem statement.

The remainder of the manuscript proceeds accordingly. The next section formalizes distributed sensing over a graph and defines the observation, latent, and communication layers of the problem. Subsequent sections introduce the latent dynamical representation, derive the variational and sequential estimation framework, analyze stability and identifiability, and present a decentralized optimization architecture with communication-efficient updates. A later section studies robustness to faults, missingness, and graph changes, followed by an evaluation protocol that clarifies how the proposed framework should be assessed under realistic operational stressors. The resulting treatment is intentionally technical and structural. The aim is not to claim that graph-based latent dynamics solves all distributed sensing problems, but to show that a carefully regularized latent operator framework offers a coherent basis for reasoning about state estimation, adaptation, and scalability in complex sensor networks.

2. Distributed Sensing over Graph-Coupled Dynamical Fields

Consider a sensor network indexed by the node set $V = \{1, \dots, N\}$, with communication or physical dependency edges $E \subseteq V \times V$. Let $G = V, E, W$ denote a weighted graph with symmetric nonnegative weight matrix W , degree matrix D , and graph Laplacian $L = D - W$. Each node i acquires observations $y_t^i \in \mathbb{R}^{m_i}$ at discrete time t , possibly with irregular availability, and may maintain a local hidden state $x_t^i \in \mathbb{R}^{n_i}$. The global hidden state is $x_t = x_t^{1\top}, \dots, x_t^{N\top\top}$, but this object is not directly observable and may itself be only a convenient discretization of an underlying field. The network is exposed to exogenous forcing a_t , local perturbations, and noise processes that need not be independent across nodes or times. The principal task is to infer coherent network-level state information and future trajectories from the distributed observations while respecting graph structure and communication constraints.

A natural starting point is a coupled state-space model. However, a naive formulation at the full state dimension is typically too expensive and too poorly conditioned for online inference. The observation process may be written as

$$y_t^i = h_t^i(x_t^i, x_t^{\mathcal{N}^i}, a_t) \quad \eta_t^i, \quad (2.1)$$

where $\mathcal{N}^i$ denotes the neighbors of node i and η_t^i is sensor noise that may include drift, quantization error, missingness effects, or adversarial corruption. The latent physical dynamics may be represented abstractly as

$$x_{t+1}^i = f_t^i(x_t^i, x_t^{\mathcal{N}^i}, a_t) \quad \xi_t^i, \quad (2.2)$$

with ξ_t^i denoting process perturbations. This form is sufficiently general to cover diffusive transport, local reaction, switching regimes, and spatially structured disturbances. Yet direct inference on x_t is undesirable because it requires estimating a large and potentially nonidentifiable nonlinear state under sparse and noisy local views.

The guiding assumption of the present work is that the network-level field admits a lower-dimensional graph-coupled latent representation $z_t \in \mathbb{R}^{rN}$, where $z_t = z_t^{1\top}, \dots, z_t^{N\top\top}$ and each node contributes a latent block $z_t^i \in \mathbb{R}^r$. The dimension r is chosen so that rN is much smaller than the effective dimension required by direct state estimation, yet large enough to preserve key dynamical modes. The observation model then factors through local or semi-local encoders and a graph-constrained state evolution. This move changes the inferential problem. Instead of attempting to reconstruct every raw component of x_t , the network seeks a latent field that supports prediction, smoothing, anomaly sensitivity, and optional decoding to physically meaningful estimates when needed [8].

A key issue in distributed sensing is that there are really two graphs in play. One graph describes physical or statistical coupling in the phenomenon being sensed, while another graph describes which devices can communicate at any given time. These need not coincide. A sensor may be physically coupled to remote regions through a transport process but only communicate with a few nearby relays. To reflect this, let G_p denote the process graph used in regularizing latent dynamics, and let G_{ct} denote the communication graph available for message passing at time t . The process graph may be fixed or slowly varying, whereas the communication graph may fluctuate due to energy-saving schedules, congestion, or environmental interference. A distributed learning system must therefore separate what it believes about the field from what it can currently communicate. The model described here uses G_p to shape latent geometry and G_{ct} to constrain the optimization and inference mechanics.

The latent dynamics are constructed to reflect both global and local mechanisms. Global structure is needed because broad system modes often span many nodes and cannot be inferred from purely local recurrences. Local structure is needed because faults, shocks, and boundary effects may remain confined. Accordingly, the latent transition is expressed as a sum of graph diffusion, low-rank transport, and adaptive residual correction:

$$z_{t+1} = \left(\alpha_0 I + \sum_{k=1}^K \alpha_k L^k \right) z_t + U_t V_t^\top z_t + B_t a_t + r_t z_t + a_t + \epsilon_t. \quad (2.3)$$

The polynomial graph term captures graph-smooth propagation or attenuation over multiple spatial scales. The low-rank factor $U_t V_t^\top$ encodes global coherent transport or long-range coupling that cannot be represented efficiently by local diffusion alone. The exogenous input operator B_t links forcing signals to latent evolution. The residual term r_t captures local nonlinear structure and regime-specific effects, while ϵ_t denotes stochastic innovation. This decomposition has practical benefits. The graph polynomial is sparse or matrix-free, the low-rank term is computationally compact, and the residual can be regularized separately to prevent destabilizing overflexibility.

Distributed sensing also imposes asymmetry between reconstruction and decision relevance. Some latent directions matter primarily because they improve predictive accuracy, while others matter because they expose anomalies, preserve connectivity-sensitive structure, or support active sensing and actuation [9]. A representation that minimizes only average observation error may collapse low-variance but strategically important modes. The graph-coupled latent construction avoids this by treating the latent field as an object of dynamical inference rather than a static compression of measurements. Each node contributes to a global latent state that is regularized over the process graph, estimated through local observations, and propagated over time by structured operators. This makes it possible to reason about observability, communication burden, and robustness using common mathematical tools.

Another subtle point concerns asynchronous and missing observations. In many distributed networks, not every node reports at every time. Some devices sleep to conserve power, some sample more frequently than others, and some intermittently fail. The present framework treats missingness explicitly through time-varying masking operators. Let M_t^i denote the local observation mask and M_t the global block-diagonal mask. Then the effective observation residual is measured not on the full reconstructed signal but on the observed subset, which prevents the inference procedure from inventing confidence in unobserved channels. This seemingly technical choice has substantial consequences because it changes how latent uncertainty is updated and how much influence dormant or faulty nodes exert on their neighbors.

Finally, distributed sensing is often performed under nonstationary environmental forcing. The underlying process may change because of seasonal effects, load shifts, maintenance events, weather conditions, or evolving usage patterns. If the latent operator is fixed, the model becomes stale. If it changes too freely, the network loses a stable coordinate system and becomes vulnerable to noise-driven drift. The operator family in the present work is therefore time-varying but regularized, allowing adaptation while preserving comparability across time. In that sense, the formal problem is not merely state estimation on a graph. It is state estimation under graph-coupled, communication-constrained, partially

observed, and temporally evolving dynamics. The next section develops the latent representation that makes this problem tractable.

3. Graph-Based Latent Dynamical Representation

The latent representation is designed to compress the distributed observation field into node-aligned coordinates whose geometry is explicitly shaped by the process graph. Let $e_{\theta_t}^i$ denote a local encoder at node i , receiving current and recent local observations, optional neighbor summaries, and exogenous context. The local latent embedding is

$$z_t^i = e_{\theta_t}^i(y_{1:t}^i, m_{1:t}^i, s_{1:t}^i, a_{1:t}), \quad (3.1)$$

where $m_{1:t}^i$ encodes missingness patterns and $s_{1:t}^i$ denotes compressed messages received from neighbors. The global latent vector is then $z_t = z_t^{1\top}, \dots, z_t^{N\top\top}$. A decoder $d_{\omega_t}^i$ reconstructs either local state surrogates or predicted measurements,

$$\hat{y}_t^i = d_{\omega_t}^i(z_t^i, z_t^{\mathcal{N}^i}). \quad (3.2)$$

This local-plus-neighbor form permits each node to decode using its own latent block together with limited graph context, thereby avoiding the need for a monolithic centralized decoder [10].

The essential modeling decision is that the latent field is not treated as an arbitrary set of node embeddings. Instead, it is endowed with a graph-sensitive quadratic form that penalizes incompatible spatial variation while preserving modes relevant to network dynamics. The basic graph regularizer is

$$\begin{aligned} \mathcal{R}_G z_t &= \sum_{i,j \in E} w_{ij} \|z_t^i - z_t^j\|_2^2 \\ &= z_t^\top L \otimes I_r z_t. \end{aligned} \quad (3.3)$$

This term encourages local latent consistency along high-weight edges. However, distributed sensor networks rarely satisfy a purely smooth field assumption. Broad trends may coexist with boundaries, anomalies, or transport corridors. Therefore, the graph penalty is applied selectively through a learned channel metric $S \succeq 0$,

$$\mathcal{R}_{GS} z_t = z_t^\top L \otimes S z_t, \quad (3.4)$$

so that some latent channels are forced to be smoother than others. Channels associated with slowly varying environmental fields may be heavily smoothed, while channels intended to capture localized disruptions may be weakly constrained.

The temporal dynamics of the latent field are modeled through a structured operator family. Rather than learning a dense transition matrix with little interpretability, the transition is decomposed as

$$A_t = \Phi L; \alpha_t \quad P_t Q_t^\top \quad \Delta_t, \quad (3.5)$$

where $\Phi L; \alpha_t$ is a graph spectral filter, $P_t Q_t^\top$ is a low-rank global coupling term, and Δ_t is a sparse or contraction-constrained residual. The graph filter may be expressed as a polynomial

$$\Phi L; \alpha_t = \alpha_{t,0} I \quad \alpha_{t,1} L \quad \cdots \quad \alpha_{t,K} L^K, \quad (3.6)$$

or, when improved spectral selectivity is required, as a rational or Chebyshev approximation. The low-rank term captures coherent modes not easily represented by localized diffusion, such as wide-area transport, periodic global forcing, or collective responses to external inputs. The residual Δ_t allows limited nonstationary correction without requiring the model to relearn the entire operator when the environment drifts.

The latent dynamics can then be written as

$$\begin{aligned} z_{t+1} &= A_t z_t + B_t a_t + g_t z_t, a_t \in \epsilon_t \\ &= (\Phi L; \alpha_t \ P_t Q_t^\top \ \Delta_t) z_t + B_t a_t + g_t z_t, a_t \in \epsilon_t, \end{aligned} \quad (3.7)$$

where g_t is a bounded nonlinear correction realized through a shallow operator network or local residual map. The separation between $A_t z_t$ and g_t is conceptually important. The structured linear component carries the dominant, analyzable spatiotemporal behavior, while the nonlinear term accounts for residual effects such as local saturation, switching, or context-dependent distortions [11]. This balance preserves flexibility without obscuring the spectrum of the transition operator.

To prevent the representation from drifting arbitrarily over time, the encoder and decoder parameters are updated under a slow variation prior,

$$\mathcal{R}_{\theta, \omega} = \sum_{t=2}^T (\|\theta_t - \theta_{t-1}\|_2^2 + \|\omega_t - \omega_{t-1}\|_2^2). \quad (3.8)$$

This regularizer does not assert that the sensing environment is stationary. Rather, it enforces coordinate continuity: when the environment changes, the latent basis should adapt gradually unless persistent evidence indicates a real shift. The benefit is that latent trajectories remain comparable across time, which is crucial for network-level diagnostics, anomaly detection, and multi-horizon forecasting. A fast-changing encoder may produce low instantaneous loss yet destroy the meaning of the latent coordinates as a state space.

The representation is further shaped by an information bottleneck principle tailored to dynamical inference. The encoder should compress nuisance variation while preserving information relevant to future observations and latent transitions. Let $q_{\varphi_t} z_t \mid \mathcal{Y}_{1:t}$ denote the approximate posterior and $p_{\rho_t} z_t$ a graph-aware reference prior. A suitable regularizer penalizes excessive divergence from the prior while rewarding predictive sufficiency:

$$\mathcal{L}_{IB} = \sum_{t=1}^T \text{KL}(q_{\varphi_t} z_t \mid \mathcal{Y}_{1:t} \parallel p_{\rho_t} z_t) - \beta I(z_t; y_{t1:T} \mid y_{1:t}). \quad (3.9)$$

In practice, the mutual-information term is approximated through predictive decoding or contrastive latent rollout objectives. The effect is to favor latent states that discard redundant measurement noise yet remain informative about future evolution.

Graph spectral structure provides a particularly useful lens for understanding the representation. Let $L = U \Lambda U^\top$ be the eigendecomposition of the Laplacian. Then graph-smooth modes correspond to low graph frequencies, while localized oscillatory patterns correspond to higher frequencies. The latent field can be expanded as

$$z_t = U \otimes I_r \hat{z}_t, \quad (3.10)$$

where $\hat{z}_t$ are graph-frequency coefficients. The graph filter $\Phi L; \alpha_t$ acts diagonally in this basis, allowing explicit control over how different spatial frequencies evolve. If higher frequencies are weakly observable or highly noise-sensitive, the transition can damp them. If the task requires anomaly sensitivity, selected high-frequency channels can be preserved or isolated. The representation thus supports a nuanced interplay between denoising and event sensitivity.

A final feature of the latent construction is that it can incorporate topology uncertainty. When the process graph is not known exactly, one may work with a nominal Laplacian $\bar{L}$ plus a perturbation δL_t , and regularize the sensitivity of the latent evolution to this perturbation. This is valuable because graph-based learning methods can otherwise be brittle to edge misspecification [12]. By making the latent operator depend smoothly on the graph and penalizing extreme sensitivity, the model acknowledges that the graph is a structural prior, not an infallible truth. In distributed sensing, this distinction is important:

communication patterns, physical couplings, and inferred adjacency relations are all imperfect proxies for the real pathways through which information or matter propagate.

The graph-based latent dynamical representation therefore serves several roles at once. It compresses distributed measurements, aligns the latent geometry with network structure, separates dominant operator behavior from residual nonlinearities, preserves coordinate continuity across time, and leaves room for topology uncertainty. These design choices prepare the ground for the inference machinery developed next, where the latent field must be estimated sequentially from partial local measurements under communication constraints.

4. Variational Inference and Sequential State Estimation

Once the latent representation is defined, the central inferential task is to estimate the posterior distribution of the latent field given distributed, noisy, and partially observed measurements. Because the model is sequential, graph-coupled, and only moderately linear, exact inference is generally intractable. The framework therefore uses a structured variational approximation that preserves local factorization where possible while allowing graph-mediated dependencies. Let $\mathcal{Y}_{1:t}$ denote all observations up to time t , together with masks and exogenous inputs. The posterior approximation takes the form

$$qz_{1:T} = qz_1 \prod_{t=1}^{T-1} qz_{t+1} \mid z_t, \mathcal{Y}_{1:t+1}, \quad (4.1)$$

with each transition factor parameterized by node-local inference networks and graph-consensus corrections. This factorization is not exact, but it provides a practical compromise between expressivity and computational structure.

The objective is a graph-aware evidence lower bound augmented by stability and communication penalties. Denote by $py_t \mid z_t$ the masked observation model and by $pz_{t+1} \mid z_t, a_t$ the latent dynamical prior. The main objective is

$$\begin{aligned} \mathcal{J} = & \sum_{t=1}^T \mathbb{E}_q \left[-\log py_t \mid z_t, M_t \right] \sum_{t=1}^{T-1} \mathbb{E}_q \left[-\log pz_{t+1} \mid z_t, a_t \right] \\ & \lambda_G \sum_{t=1}^T \mathbb{E}_q \left[z_t^\top L \otimes Sz_t \right] \lambda_T \sum_{t=2}^{T-1} \|A_t - A_{t-1}\|_F^2 \\ & \lambda_Q \sum_{t=1}^T \text{KL}(qz_t \mid \mathcal{Y}_{1:t} \parallel p_{\rho_t} z_t) \lambda_C \mathcal{C}_{\text{comm}}. \end{aligned} \quad (4.2)$$

The first term measures reconstruction of observed components only. The second enforces consistency with the latent transition prior. The third shapes the geometry of the latent field over the graph. The fourth discourages abrupt changes in the transition operator sequence. The fifth regularizes posterior concentration. The last term captures communication cost, which may be modeled as expected message dimension, quantization burden, or edge activation penalty depending on the deployment scenario.

For the local observation term, masked reconstruction is essential [13]. If node i reports only a subset of channels at time t , the residual is evaluated as

$$\ell_t^i = \|M_t^i (y_t^i - \hat{y}_t^i)\|_{\Sigma_{t,i}^{-1}}^2 \log |\Sigma_{t,i}|, \quad (4.3)$$

where $\Sigma_{t,i}$ is a learned or structured observation covariance on the active channels. This formulation permits heteroskedastic uncertainty and prevents the model from treating missing values as informative zeros or implicit imputations. When certain sensors are absent for long periods, their uncertainty naturally expands and is then reduced only through graph-coupled information arriving from neighbors or through future direct observations.

Sequential estimation proceeds through alternating local updates and graph-consensus corrections. Each node first computes a provisional local posterior using its own recent measurements and previous latent estimate. Let $\hat{q}_t^i$ denote this provisional factor. The node then exchanges compressed summaries

with neighbors, typically means and low-rank covariance sketches, and updates its posterior via a graph-conditioned fusion step. The latent mean update can be written abstractly as

$$\begin{aligned} \mu_t^i = \arg \min_{\mu} \quad & \ell_t^i \mu \|\beta_t^i\| \mu - \bar{\mu}_{t|t-1}^i \|^2_{P_{t|t-1}^{i-1}} \\ & \gamma_t^i \sum_{j \in \mathcal{N}_{ci,t}} w_{ij}^c \|\mu - \Pi_{ij} \mu_t^j\|_2^2, \end{aligned} \quad (4.4)$$

where $\bar{\mu}_{t|t-1}^i$ is the predicted latent mean from the dynamical prior, $P_{t|t-1}^i$ is the corresponding covariance approximation, $\mathcal{N}_{ci,t}$ denotes neighbors in the communication graph, and Π_{ij} is a possible alignment or projection operator when local latent frames are partially node-specific. The third term is a consensus penalty that pushes neighboring posteriors toward agreement on shared latent content without forcing identical local states.

The predicted latent mean itself is obtained from the structured operator,

$$\bar{\mu}_{t1|t} = A_t \mu_t \ B_t a_t \ \mathbb{E}_q g_t z_t, a_t, \quad (4.5)$$

while covariance propagation follows a linearized approximation,

$$P_{t1|t} = F_t P_{t|t} F_t^\top \ Q_t, \quad (4.6)$$

where F_t is the Jacobian of the latent transition around the current posterior mean and Q_t is the innovation covariance. Because the latent dimension is moderate and the operator is structured, one can compute or approximate this propagation efficiently using low-rank updates and matrix-free products. In decentralized settings, full covariance transmission is rarely feasible, so nodes communicate diagonal, block-diagonal, or low-rank covariance summaries depending on bandwidth availability.

A practical complication is that real sensor data often contain outliers, abrupt glitches, or delayed packets. Standard quadratic losses can overreact to such irregularities, especially when the model is allowed to adapt online. For this reason, the inference objective can be robustified by replacing the quadratic observation and transition residuals with pseudo-Huber or Student- t penalties. For a generic residual r , the pseudo-Huber function is

$$\phi_\delta r = \delta^2 \left(\sqrt{1 + \|r\|_2^2 / \delta^2} - 1 \right). \quad (4.7)$$

This behaves quadratically near zero and linearly for large discrepancies, preventing a single corrupted measurement from inducing disproportionate changes in the posterior or operator estimates. In distributed networks this is particularly valuable because a local sensor fault can otherwise spread misleading information through consensus updates.

The variational framework also supports fixed-lag smoothing in addition to filtering. Instead of estimating z_t from data up to time t only, the algorithm may maintain a lag window H and re-estimate $z_{t-H+1:t}$ as new measurements arrive. This substantially improves state reconstruction in slowly observed or partially missing networks because future neighbor observations often disambiguate earlier latent uncertainty [14]. The cost is increased communication and memory, but the structured operator form keeps the smoothing problem manageable. In many deployments, a modest lag yields a favorable trade-off between real-time responsiveness and estimation quality.

Another important aspect concerns the online update of the operator family itself. The transition matrices A_t and input operators B_t are not fixed; they must adapt as the environment changes. However, updating them too aggressively during filtering risks conflating state uncertainty with genuine drift. The present framework therefore separates fast state inference from slower operator adaptation. Latent states are updated at each step, whereas operator parameters are adjusted over mini-batches or change-sensitive windows using accumulated sufficient statistics. This two-timescale design is crucial for preserving latent coordinate continuity while remaining responsive to persistent structural changes.

The result is a sequential inference system that integrates local sensing, graph-aware posterior fusion, robust residual handling, fixed-lag smoothing, and controlled operator adaptation. Importantly, this is not merely a decentralized implementation of a central Kalman-style model. The graph structure shapes the latent space, the communication cost enters the objective explicitly, and the inference process is designed around compressed message exchange rather than unrestricted central aggregation. These features determine how well the model can scale across the network and remain stable under realistic sensing conditions.

5. Spectral Stability, Identifiability, and Error Propagation

The value of a latent dynamical representation depends not only on how well it reconstructs local observations but also on whether its repeated application remains stable and interpretable. In distributed sensor networks, instability is especially damaging because recursive prediction errors can interact with communication delays and missing observations to produce network-wide inconsistency. For this reason, the transition operator family is analyzed through its spectral behavior, perturbation sensitivity, and induced error dynamics. The key object is the effective transition Jacobian,

$$F_t = A_t \frac{\partial g_t}{\partial z} z_t, a_t, \quad (5.1)$$

which governs first-order propagation of latent perturbations.

Let $e_t = z_t - \hat{z}_t$ denote estimation error. Under local linearization and bounded innovation mismatch ν_t , the error evolves as

$$e_{t+1} = F_t e_t + \nu_t. \quad (5.2)$$

Consequently,

$$\|e_{t+1}\|_2 \leq \|F_t\|_2 \|e_t\|_2 + \|\nu_t\|_2. \quad (5.3)$$

If $\sup_t \|F_t\|_2 < 1$, then the error process is contractive and cumulative error remains bounded under persistent bounded perturbations [15]. This condition is sufficient rather than necessary, but it is operationally meaningful because it links long-horizon stability to the spectral radius and gain of the learned operator family. When $\|F_t\|_2$ approaches or exceeds one in poorly observed directions, even small local estimation errors can amplify and contaminate remote regions through graph coupling. Thus spectral regularization is not just a training convenience; it directly affects the viability of network-level inference.

To control this behavior, the model applies shrinkage or clipping to the singular values of the learned transition. Let the singular value decomposition of A_t be $A_t = U_t \Sigma_t V_t^\top$. The filtered operator is

$$\tilde{A}_t = U_t \rho \Sigma_t V_t^\top, \quad (5.4)$$

where ρ is a spectral shaping function satisfying $\rho\sigma \leq \sigma$ for unstable or weakly supported modes. One convenient choice is

$$\rho\sigma = \frac{\sigma}{1 + \kappa\sigma^2}, \quad (5.5)$$

which dampens large singular values more strongly than moderate ones. Another option is hard radius clipping,

$$\rho\sigma = \min\{\sigma, \tau\}. \quad (5.6)$$

The choice depends on whether the application prefers gradual attenuation or explicit spectral bounds. In either case, the aim is to prevent the model from encoding excessive transient amplification in directions that the data do not reliably constrain.

Identifiability in graph-based latent models is more subtle than in conventional state-space systems because the latent coordinates can often be transformed without changing the likelihood. If S is an invertible block transformation, then $z'_t = Sz_t$, $A'_t = SA_tS^{-1}$, and appropriately transformed encoders and decoders may yield equivalent observation fit. To reduce this ambiguity, the framework anchors the latent space through graph regularization, temporal continuity, and channel-wise smoothness structure. The graph penalty prefers latent fields aligned with the process topology, while slow evolution of encoder parameters discourages arbitrary reorientation of the latent basis from one time step to the next. These constraints do not eliminate all invariances, but they reduce the space of equivalent representations enough to make the learned dynamics comparable over time.

Observability also plays a central role. A latent direction is practically meaningless if it cannot be recovered from distributed observations over a reasonable horizon. Let C_t denote the linearized observation operator around the current posterior mean. Over horizon H , the empirical observability matrix is

$$\mathcal{O}_{t,H} = \begin{bmatrix} C_t \\ C_{t1}F_t \\ C_{t2}F_{t1}F_t \\ \vdots \\ C_{tH-1}F_{tH-2} \cdots F_t \end{bmatrix}. \quad (5.7)$$

If $\sigma_{\min}\mathcal{O}_{t,H}$ is small, then certain latent directions are weakly observable even if short-term reconstruction appears acceptable. In a distributed network, this weakness may arise because relevant information is fragmented across nodes and only becomes available after several communication rounds [16]. The graph-based latent design helps by concentrating information into spatially coherent modes and by avoiding latent directions that are dynamically active yet observationally unsupported. This does not guarantee perfect observability, but it improves conditioning compared with unconstrained embeddings.

The graph spectrum itself provides a useful decomposition of stability. Since the graph filter $\Phi L; \alpha_t$ acts diagonally in the Laplacian eigenbasis, each graph frequency evolves under a mode-specific gain. For eigenvalue λ_ℓ , the graph-filter contribution is $\phi_t \lambda_\ell$. If high-frequency graph modes correspond primarily to noise or missingness artifacts, then $|\phi_t \lambda_\ell|$ should be small. If localized anomalies are important, selected higher frequencies may remain active but should be controlled by observational support and residual regularization. One can therefore define a graph-spectral stability margin,

$$\gamma_t \lambda_\ell = |\phi_t \lambda_\ell| \|\Pi_\ell (P_t Q_t^\top \Delta_t) \Pi_\ell\|_2, \quad (5.8)$$

where Π_ℓ projects onto the ℓ -th graph-frequency subspace. This quantity helps diagnose whether instability is concentrated in broad network modes or in localized oscillatory directions.

Perturbation sensitivity with respect to graph misspecification is another critical concern. Suppose the nominal Laplacian is $\bar{L}$ but the true coupling operator is $\bar{L} + \delta L$. Then the transition perturbation induced by a polynomial graph filter is

$$\delta A_t = \sum_{k=1}^K \alpha_{t,k} (\bar{L} + \delta L)^k - \bar{L}^k. \quad (5.9)$$

For small δL , first-order expansion yields

$$\delta A_t \approx \sum_{k=1}^K \alpha_{t,k} \sum_{s=0}^{k-1} \bar{L}^s \delta L \bar{L}^{k-1-s}. \quad (5.10)$$

The induced operator norm can then be bounded in terms of $\|\delta L\|_2$ and the polynomial coefficients. This reveals a trade-off. High-order graph filters can represent rich multiscale interactions, but their sensitivity to graph error may grow quickly if not controlled. Hence the latent model should prefer spectrally smooth filters or explicitly penalize graph-derivative sensitivity when topology uncertainty is substantial.

Another source of instability is time variation in the operator family. Even if each A_t is individually stable, rapid alternation between nearby unstable directions can produce significant transient growth. For this reason, the regularization term $\|A_t - A_{t-1}\|_F^2$ is more than a smoothness prior. It reduces the chance of oscillatory operator estimates that exploit local residual minimization while producing physically implausible temporal switching [17]. In practice, such oscillation is easy to miss if one evaluates only one-step forecast error. Spectral diagnostics and rollout experiments are needed to detect it.

Finally, error propagation in distributed networks interacts with communication noise and delayed consensus. If local posterior means are fused through imperfect messages, the effective error dynamics become

$$e_{t+1} = F_t e_t + H_t \delta_t + \nu_t, \quad (5.11)$$

where δ_t denotes communication-induced discrepancies and H_t maps those discrepancies into the latent update. Stability then depends jointly on the transition gain and the consensus mechanism. A stable latent operator can still produce poor network estimates if the communication layer injects inconsistent neighbor information faster than local filtering can absorb it. This is why communication-aware optimization is studied in the next section together with the dynamical model itself. In a distributed sensor network, stability is never purely temporal or purely spatial; it emerges from the interaction between latent dynamics, observability, and message exchange.

6. Decentralized Optimization and Communication-Efficient Learning

A graph-based latent model is only operationally useful if it can be trained and updated without requiring constant full centralization of data. In many sensor networks, edge devices can compute local statistics but cannot stream all raw measurements continuously. Power limitations, privacy restrictions, intermittent connectivity, and latency constraints make centralized optimization unrealistic or undesirable. The learning architecture therefore has to mirror the network structure. This leads to a decentralized optimization problem in which local variables, shared edge variables, and global operator summaries are updated through constrained consensus.

Let Θ^i denote the local parameters maintained by node i , including encoder weights, local decoder components, and local blocks of latent sufficient statistics. Let Ξ denote shared operator parameters such as graph filter coefficients, low-rank factors, and selected global hyperparameters. A generic distributed objective can be written as

$$\min_{\{\Theta^i\}, \Xi} \sum_{i=1}^N \mathcal{L}^i(\Theta^i, \Xi) + \lambda_{\text{cons}} \sum_{i,j \in E_c} \Psi_{ij}(\Theta^i, \Theta^j), \quad (6.1)$$

where E_c denotes communication edges and Ψ_{ij} measures disagreement between neighboring local models or latent summaries. Depending on the system design, Ξ may be stored centrally, redundantly at cluster heads, or maintained through distributed averaging. The key requirement is that most optimization steps should be executable using local data and neighbor messages only.

An effective strategy is consensus ADMM applied to the latent sufficient statistics and selected parameter blocks. For each communication edge i, j , introduce auxiliary variables v_{ij} enforcing agreement

on shared quantities. The augmented Lagrangian then takes the form

$$\mathcal{L}_{\text{aug}} = \sum_{i=1}^N \mathcal{L}^i(\Theta^i, \Xi)_{i,j \in E_c} \left[\frac{\rho}{2} \|K_{ij} \Theta^i - v_{ij} u_{ij}\|_2^2 + \frac{\rho}{2} \|K_{ji} \Theta^j - v_{ij} w_{ij}\|_2^2 \right], \quad (6.2)$$

where K_{ij} extracts the parameter or latent components that should agree, and u_{ij}, w_{ij} are scaled dual variables. Each node solves a local update using its own observations and current neighbor summaries, then exchanges only the minimal sufficient information needed for the consensus step [18]. This architecture is well suited to graph-based latent models because the graph already determines which nodes should coordinate most strongly.

Communication efficiency is not merely a systems concern; it influences the statistical properties of learning. If messages are too aggressively compressed, the consensus process may introduce bias or lag, especially in fast-changing environments. If messages are too detailed, the network may exceed bandwidth budgets or energy constraints. The framework accounts for this by representing messages in the latent space rather than in raw observation space. Since the latent dimension r is moderate and structured, transmitting local posterior means, selected covariance diagonals, or low-rank sketches is far cheaper than transmitting full measurements. A generic communication budget term may be expressed as

$$\mathcal{C}_{\text{comm}} = \sum_{t=1}^T \sum_{i,j \in E_c} c_{ij}(b_{t,ij}, q_{t,ij}), \quad (6.3)$$

where $b_{t,ij}$ denotes the number of transmitted latent coefficients and $q_{t,ij}$ represents quantization precision. Penalizing this term in the objective encourages compact messages and selective edge activation.

Compression can be performed through learned projections. Let R_{ij} be a low-dimensional projection matrix tailored to edge i, j . Then node i transmits

$$m_{t,ij} = R_{ij}^\top \mu_t^i, \quad (6.4)$$

possibly together with a covariance sketch. The receiving node uses a reconstruction or compatibility map to integrate the message into its local posterior update. In principle, these projections can be adapted to maximize mutual information under bandwidth limits, favoring latent channels that are most informative for neighboring nodes. This is particularly useful in heterogeneous graphs where some edges connect highly correlated regions and others serve only occasional routing functions.

The optimization of global operator parameters can also exploit structure. The graph filter coefficients α_t are low-dimensional and can be estimated from aggregated sufficient statistics rather than raw data. The low-rank factors P_t and Q_t can be updated using distributed matrix sketching, with each node contributing local cross-covariance terms. Specifically, if $S_t = \sum_i S_t^i$ and $T_t = \sum_i T_t^i$ are distributed moment accumulators, then a rank- k update may be obtained from a sketched eigenproblem rather than from dense centralized regression. This preserves scalability as the number of nodes grows.

A crucial design choice is the timescale separation between local latent updates and global parameter synchronization. Full synchronization at every step is often unnecessary and expensive [19]. Instead, the network may perform several rounds of local filtering and neighbor fusion before triggering a slower operator update. Such staggered schedules exploit the fact that latent states change at every time step, whereas structural operator drift is usually slower. However, too much delay can cause local models to diverge or overfit recent data. The framework therefore ties synchronization frequency to residual diagnostics and estimated drift magnitude. When the network detects persistent mismatch between predicted and observed latent transitions, global parameter sharing becomes more aggressive. During stable periods, synchronization can be sparse.

One of the main benefits of the graph-based latent representation is that it admits graceful degradation under communication loss. Since each node maintains a local latent dynamics model and receives only compressed neighbor summaries, temporary edge failures do not necessarily collapse inference. The local posterior simply becomes more uncertain, and the dynamical prior carries more weight until communication resumes. This is preferable to architectures whose predictive quality depends critically on dense synchronous message passing at every step. In practice, robustness to sparse or delayed communication is often more valuable than marginal gains in ideal centralized accuracy.

From a computational perspective, the complexity of local updates is dominated by encoder evaluation, latent covariance propagation, and small-scale optimization over local posterior means. The graph filter component can be applied matrix-free using sparse Laplacian products, while the low-rank correction costs scale with the chosen rank k . Consensus updates scale with the number of active communication edges and the compressed message size. This yields a modular computational profile in which cost can be traded against fidelity by adjusting latent dimension, rank, message precision, and synchronization frequency. Such explicit trade-offs are important because distributed sensor networks operate under hard resource budgets rather than abstract asymptotics.

The decentralized optimization architecture therefore complements the statistical model rather than merely implementing it. By embedding consensus, compression, and synchronization into the learning objective, the framework recognizes that what can be inferred is inseparable from how information moves through the network. In distributed sensing, the representation must be not only accurate and stable, but also communicable.

7. Robustness to Missing Data, Topology Changes, and Sensor Faults

Sensor networks in realistic environments are rarely clean or static. Nodes fail, links disappear, clocks drift, packets arrive late, calibration shifts, and local conditions can render a subset of sensors temporarily unreliable [20]. A latent model that assumes complete measurements and fixed topology may perform well in simulation while failing in deployment. Robustness must therefore be designed into both the latent representation and the inference procedure. The present framework approaches this by treating missingness, topology perturbation, and fault contamination as structured sources of uncertainty rather than as exceptional corner cases.

Missing data are handled through explicit observation masks and uncertainty inflation. Let M_t be the block-diagonal mask that selects active observation channels. The observation likelihood then depends only on $M_t y_t$, while unobserved channels are excluded from the reconstruction residual. This avoids biased imputations, but it also means that latent uncertainty can expand in poorly observed regions of the graph. To prevent this uncertainty from spreading uncontrollably, the graph-coupled prior provides a regularized borrowing of strength from neighboring nodes. The filtered posterior is therefore shaped by a tension between local evidence and graph-constrained propagation. When a node is unobserved for an extended interval, its latent state evolves mostly under the dynamical prior and neighbor-consensus information, leading to larger posterior variance but not necessarily complete loss of coherence.

Faulty observations are more problematic than missing ones because they actively push the posterior in the wrong direction. To handle this, the model introduces an additive sparse corruption term o_t in the observation equation,

$$y_t = \hat{y}_t + o_t + \eta_t, \quad (7.1)$$

and penalizes o_t with a sparsity-promoting norm,

$$\mathcal{R}_o = \lambda_o \sum_{t=1}^T \|o_t\|_1. \quad (7.2)$$

This encourages the inference procedure to explain persistent structured patterns through the latent dynamics, while isolating abrupt channel-specific deviations as faults or anomalies. In nodewise form,

the penalty can be weighted according to sensor reliability priors so that historically unstable channels are more readily attributed a nonzero fault term. This is especially useful in heterogeneous networks where different device classes exhibit different noise and failure characteristics.

Topology variation is handled at both the process and communication levels. If the process graph changes slowly due to physical reconfiguration or shifting correlation structure, the latent operator can be updated by allowing the Laplacian to evolve under a regularized perturbation model,

$$L_t = L_{t-1} + \Delta L_t, \quad (7.3)$$

with penalty

$$\mathcal{R}_L = \lambda_L \sum_{t=2}^T \|\Delta L_t\|_F^2. \quad (7.4)$$

This does not imply that the graph is fully relearned from data at every step [21]. Rather, it allows modest topology adaptation when persistent residual patterns indicate that the nominal graph no longer explains the observed latent coupling. For communication topology G_{ct} , adaptation is more operational. The inference algorithm must continue under changing edge availability, which it does by using whatever neighbors are present at each time and inflating uncertainty when consensus information is sparse.

The interaction between topology uncertainty and latent dynamics can be analyzed through the sensitivity of the graph filter to Laplacian perturbations. Because the graph filter is regularized to avoid excessive high-order sensitivity, small topology changes do not produce catastrophic operator shifts. In addition, the low-rank component of the transition can absorb some broad changes in interregional coupling even when the local graph is imperfect. This decomposition is valuable in practice because many network perturbations affect only a subset of edges, while the dominant global transport structure remains approximately intact.

Robustness also requires credible uncertainty quantification. A deterministic latent forecast may appear stable even when confidence should be low due to missing nodes or conflicting neighbor messages. The framework therefore propagates posterior covariance or its structured approximation alongside the latent mean. In settings where exact covariance propagation is too costly, one can retain diagonal or low-rank covariance factors sufficient to reflect directional uncertainty. A useful nodewise uncertainty score is

$$u_t^i = \text{tr}(P_t^i) + \chi_{j \in \mathcal{N}_{ci}, t} \|\mu_t^i - \Pi_{ij} \mu_t^j\|_2^2, \quad (7.5)$$

where the first term measures local posterior variance and the second measures disagreement with neighbors. Elevated values of u_t^i signal either poor local observability or communication-induced inconsistency. Such scores are useful for active sensing policies, maintenance prioritization, or anomaly triage.

Nonstationary environmental forcing introduces a different type of robustness challenge. When the environment drifts, the model must adapt without interpreting every unusual measurement as a structural change. The present framework uses residual persistence to distinguish shift from noise. Let

$$r_t = z_{t1} - A_t z_t - B_t a_t - g_t z_t, a_t. \quad (7.6)$$

Rather than updating the operator immediately from a large $\|r_t\|$, the algorithm accumulates evidence over a short horizon and evaluates whether the residual pattern is consistent across neighboring nodes and graph frequencies. Persistent low-frequency residuals suggest genuine process drift. Isolated high-frequency bursts are more consistent with local faults or transient shocks. This graph-frequency perspective helps stabilize online adaptation by exploiting the spatial organization of mismatch [22].

Another important scenario is partial adversarial contamination, where a subset of sensors or communication edges is manipulated. Full adversarial robustness is beyond the scope of the present formulation,

but the graph-aware latent design offers partial protection. Sparse corruption modeling, robust loss functions, and limited trust in any single neighbor message all reduce the leverage of compromised nodes. Furthermore, consensus penalties can be reweighted by reliability scores so that historically inconsistent or suspicious nodes exert less influence on their neighbors. Since the latent state is low-dimensional and structured, abnormal influence patterns are easier to detect than in raw high-dimensional observation space.

The broader point is that robustness in distributed sensor networks is not a single mechanism. It emerges from several coordinated design choices: masking and uncertainty-aware inference for missingness, sparse corruption variables for faults, graph-perturbation regularization for topology change, covariance propagation for confidence tracking, and residual-persistence logic for nonstationary adaptation. A model lacking any one of these may still work in benign conditions, but its failure modes will become visible under operational stress. The proposed framework treats those stressors as integral to the learning problem rather than as afterthoughts.

8. Evaluation Protocol and Analytical Performance Criteria

A rigorous assessment of graph-based latent dynamics for distributed sensor networks must go beyond aggregate one-step prediction error. Since the model is designed for partial observability, sequential estimation, communication-constrained operation, and robustness under drift, evaluation should probe each of those axes explicitly. The present section develops an analytical protocol for such assessment. Although the exact datasets and numerical values depend on the application domain, the protocol itself is general and can be used to test whether the learned latent representation remains coherent, stable, and useful under realistic network stressors.

The starting point is a synthetic or semi-synthetic generator defined over a known graph. The hidden field should contain multiple spatial and temporal scales: graph-smooth background trends, low-rank global oscillations or transport, localized shocks, intermittent sensor faults, and slowly drifting regime parameters. Exogenous inputs may modulate a subset of nodes or drive broad field changes. Missingness patterns should include random dropout, scheduled sleeping, and bursty communication outages. Such a generator makes it possible to separate the difficulty of latent reconstruction from the difficulty of network communication and adaptation. Real datasets can then be used as complementary tests, but controlled generators remain important because they expose identifiability and stability issues that may be obscured by unknown ground truth.

The first group of metrics concerns latent and observational fidelity [23]. Let $\hat{z}_t$ denote the posterior mean and $\hat{y}_t$ the decoded observation estimate. Useful statistics include filtered state error $\|z_t - \hat{z}_t\|_2$, fixed-lag smoothed state error, masked reconstruction error on observed channels, and imputation error on held-out channels. Because the latent field is graph-structured, one should also examine graph-frequency reconstruction. If $L = U\Lambda U^\top$, then the graph-frequency error is

$$\mathcal{E}_{\text{freq}} = \frac{1}{T} \sum_{t=1}^T \|U^\top \otimes I_r z_t - \hat{z}_t\|_{W_\Lambda}^2, \quad (8.1)$$

where W_Λ weights frequencies according to task relevance. This metric reveals whether the model reconstructs broad smooth modes at the expense of localized events or vice versa.

The second group of metrics addresses dynamical stability. Since many failures only appear under recursive rollout, the model should be evaluated at multiple horizons. One may compute

$$\mathcal{E}_H = \frac{1}{T-H} \sum_{t=1}^{T-H} \|z_{t+H} - \hat{z}_{t+H|t}\|_2^2, \quad (8.2)$$

where $\hat{z}_{t+H|t}$ is the H -step latent forecast from time t . Growth of $\mathcal{E}_H$ with H reveals whether the learned operator accumulates error benignly or explosively. Complementary diagnostics include empirical spectral radius, maximum singular value of the local transition Jacobian, and graph-spectral gain profiles.

These are particularly informative around regime changes, where short-term fit may remain good while rollout stability degrades.

A third group of metrics measures adaptation under nonstationarity. When the generator changes regime at time t^* , the evaluation should quantify detection delay, adaptation transient, and post-shift steady-state error. Let s_t be a regime indicator and $\hat{s}_t$ a shift score derived from residual persistence or operator updates. Then one can measure the delay between t^* and the time at which the model settles into acceptable forecast error for the new regime. Equally important is false adaptation during stationary periods, which reflects how often noise or local anomalies cause unnecessary operator drift. A robust model should adapt promptly to persistent shifts but remain inertial in the face of isolated corruption.

Communication-aware performance must also be measured explicitly. If the model is evaluated only under full message availability, one cannot tell whether the distributed architecture is actually useful. Let B denote a communication budget in transmitted latent coefficients per step. For each B , evaluate state error, forecast error, consensus discrepancy, and energy or latency proxies. The resulting rate-distortion style curve reveals how gracefully the model degrades as bandwidth decreases. A useful metric is

$$\mathcal{D}_{\text{cons}} = \frac{1}{T} \sum_{t=1}^T \sum_{i,j \in E_c t} w_{ij}^c \|\mu_t^i - \Pi_{ij} \mu_t^j\|_2^2, \quad (8.3)$$

which quantifies inconsistency across neighboring posteriors [24]. Low $\mathcal{D}_{\text{cons}}$ under moderate communication compression suggests that the latent representation remains communicable as intended.

Robustness to faults and missingness requires separate evaluation. For missing data, one should vary dropout rate, burst length, and spatial concentration of missing nodes. Performance should be reported not only on active nodes but also on recovered estimates for dormant ones. For sensor faults, one should vary fault sparsity, amplitude, and temporal persistence. The sparse corruption term can then be assessed through detection metrics as well as through its impact on latent stability. A model that detects faults but destabilizes neighboring inference is not operationally satisfactory. Uncertainty calibration is also essential here: when observations vanish or conflict, posterior confidence should widen appropriately. Calibration can be examined through empirical coverage of predictive intervals or through proper scoring rules adapted to masked multivariate observations.

The choice of baselines matters. Suitable comparators include nodewise recurrent models without graph coupling, graph neural predictors without explicit latent-state inference, classical distributed Kalman-like models with fixed operators, and centralized latent models without communication constraints. These baselines expose the contribution of each structural component. For example, comparison against a centralized model reveals the cost of decentralization, while comparison against a graph predictor without latent operator structure reveals the value of explicit dynamical regularization. Because the present framework integrates multiple design principles, ablation studies are particularly informative. Removing graph regularization, low-rank transport, slow operator adaptation, or communication penalties in turn clarifies which features are responsible for stability, efficiency, or robustness.

A final and often neglected part of evaluation concerns structural diagnostics. Since the framework is designed around operator behavior, it should be judged using operator-level criteria. These include sparsity or rank of adaptive updates, smoothness of operator trajectories over time, alignment of active latent modes with graph frequencies, and sensitivity of forecasts to graph perturbation. Such diagnostics are not merely interpretive. They indicate whether the model’s internal geometry is consistent with the claimed design principles. In distributed sensing, a system may achieve acceptable average error for the wrong structural reasons, only to fail when conditions shift [25]. Operator diagnostics help reveal such hidden fragility.

The overall evaluation protocol is therefore intentionally multidimensional. It treats forecasting, estimation, adaptation, communication, robustness, and structure as interdependent rather than interchangeable. This is necessary because the proposed model is meant for deployment in environments where failure rarely appears as a simple increase in one-step loss. More often, failure appears as delayed

adaptation, unstable rollout, overconfidence under missing data, or intolerable communication burden. A convincing assessment must therefore expose all of these dimensions.

9. Conclusion

This paper developed a graph-based latent dynamical framework for distributed sensor networks under partial observability, communication constraints, and nonstationary environmental forcing. The central premise has been that reliable sensing at network scale requires more than local prediction accuracy. It requires a latent representation whose geometry reflects graph structure, a dynamical operator whose spectrum can be controlled and interpreted, an inference procedure that can fuse incomplete local information sequentially, and an optimization architecture that respects the realities of decentralized communication. The proposed formulation addressed these requirements by combining graph-regularized latent variables, structured transition operators, robust variational filtering and smoothing, communication-aware consensus updates, and explicit treatment of missingness, faults, and topology change.

Across the technical development, several themes recurred. First, the graph should shape the latent space rather than merely serve as a message-passing scaffold. Second, operator structure matters because stability, identifiability, and long-horizon performance depend on how latent modes evolve under repeated application. Third, decentralized computation is inseparable from statistical modeling in sensor networks, since bandwidth limits and intermittent connectivity constrain what can be inferred in practice. Fourth, robustness must be built from multiple mechanisms, including masking, sparse fault modeling, uncertainty propagation, and controlled operator adaptation. The paper therefore framed distributed sensing as a unified problem of representation, dynamics, and network-aware computation.

The resulting perspective does not eliminate the inherent difficulty of learning in real sensor networks. Model bias, graph uncertainty, heterogeneous devices, and severe regime shifts remain challenging. Even so, a graph-based latent dynamical approach offers a principled basis for organizing those challenges. By making latent geometry, operator behavior, and communication cost explicit objects of design, it becomes possible to analyze trade-offs among scalability, robustness, and dynamical fidelity with considerably greater clarity than is available in purely local or purely black-box approaches. In that sense, the framework provides a technical foundation for future work on adaptive networked inference, active sensing, and resilient distributed learning [26].

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